AN

INTRODUCTION TO THE MENSURATION OF SUPERFICIES and SOLIDS.

3

For the Uſe of Learners.

By WILLIAM PAYNE.

K

LONDON:
Printed by H. Hart, Popping's Court, Fleetſtreet.
Sold by T. Payne, at the Mews Gate; and
M. Hingeston, at Temple Bar.
MDCCLXVIII.

ADVERTISEMENT.

THIS Treatiſe is intended for an additional Book, to the Second Edition of the Author's Introduction to GEOMETRY, *firſt publiſhed in* 1767, *and the Reader is referred to it in the Reaſon given for each Rule.*

BRITISH MUSEUM

BOOK VII.

Of MEASURING.

338. *LINES, lengths, or dimenſions, are eſtimated or meaſured by inches, feet, yards, &c.*

339. *A ſquare, whoſe ſide is in length one inch, one foot, one yard, &c. is called a ſquare inch, ſquare foot, ſquare yard, &c.*

340. *A cube, whoſe ſide is in length one inch, one foot, one yard, &c. is called a cubic inch, cubic foot, cubic yard, &c.*

341. *Surfaces are eſtimated or meaſured by the number of ſquare inches, feet, yards, &c. which they contain.*

342. *Solids are eſtimated or meaſured by the number of cubic inches, feet, yards, &c. which they contain.*

Problem 1.

343. *To multiply feet, inches and parts, by feet, inches and parts, which method is termed croſs multiplication.*

RULE.

Set the feet in the multiplier, under the leaſt denomination in the multiplicand, and the reſt in order; multiply as in common arithmetick, divide each product by 12 *(as you go on) place the firſt remainder under the multiplying figure, and the reſt in order, adding the ſeveral quotients to the next ariſing products; and having thus finiſhed the multiplication, the ſum of all will be the product required.*

EXAMPLE.

What is the product of 7 feet, 6 inches, 9 parts, by 6 feet, 5 inches, 3 parts?

f. ′ ″
Now the multiplicand is 7 : 6 : 9
- - - the multiplier is 6 : 5 : 3 duly placed

1 : 10 : 8 : 3
3 : 1 : 9 : 9 :
45 : 4 : 6 :

The product is - - 48 : 8 : 2 : 5 : 3

EXAMPLE 2.

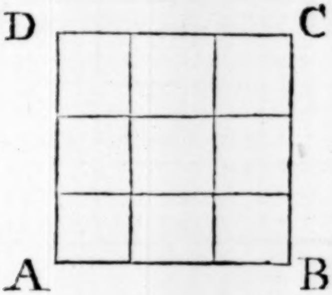
f. i. ″ f. i. ″
Multiply - - 9 : 7 : 3 - - by 7 : 9 : 3
The multiplier is - - 7 : 9 : 3 properly placed

2 : 4 : 9 : 9
7 : 3 : 5 : 3 :
67 : 2 : 9 :

The product is 74 : 8 : 7 : 0 : 9

Problem 2.

344. *To find the area, or content of a ſquare* ABCD.



RULE.

Multiply the length of the ſide by itſelf, and the product will expreſs the area.

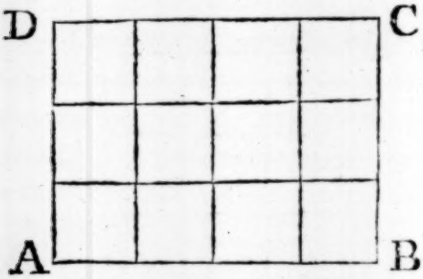
For if two adjacent ſides of the ſquare AB, AD are divided into the ſame number of equal parts, and from the points of diviſion lines be drawn parallel to the other ſides AD DC; it is evident that the given ſquare will be divided into leſſer ſquares, that the number of little ſquares in a row is equal to the number of parts (or meaſures) in the ſide, that the number of rows is likewiſe equal to the number of parts in the ſide, and conſequently, the ſquare of that number will expreſs the area.

EXAMPLE.

Let the ſide AB be three meaſures of any kind; then $AB \times AB = 3 \times 3 = 9$ the area.

Problem 3.

345. *To find the area of a rectangle or oblong* ABCD.



RULE.

Multiply the length by the breadth, namely, the baſe by the perpendicular; and the product will expreſs the area.

For if the baſe AB, and perpendicular AD, are divided into parts of equal length, and parallels be drawn to the other ſides AD, DC, from the points of diviſion, it is evident that the rectangle is divided into little ſquares, that the number in a row upon the baſe, is equal to the number of parts or meaſures in it, the number of rows is equal to the number of parts in the perpendicular, and conſequently the product of thoſe numbers will expreſs the area.

EXAMPLE.

Let the baſe AB be 4, and the perpendicular BC be 3. Then $AB \times BC = 4 \times 3 = 12$, the area.

EXAMPLE

EXAMPLE 2.

What is the area of a rectangular floor, whoſe length is 33 feet 9 inches, and breadth is 22 feet 6 inches?

1. By croſs multiplication.

f. i.
The length or baſe is - - 33 : 9
The breadth or perpendicular is - - : 22 : 6

16 : 10 : 6
742 : 6

The area is - - - 759 : 4 : 6

2. By decimals.

The length or baſe is - - 33,75
The breadth or perpendicular is 22,5 ×

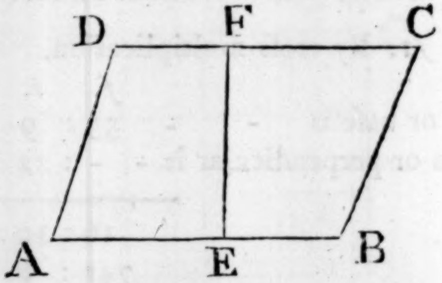
16875
6750
6750

The product is - - - 759,375. the area as before.

Problem

Problem 4.

346. *To find the area of a parallelogram* ABCD.



RULE.

Multiply the length by the breadth, and the product will give the area.

Becauſe a parallelogram is equal to a rectangle of the ſame baſe and altitude - - - - 94

EXAMPLE.

Let the length or baſe AB be 242 yards, and the breadth or perpendicular EF be 160 yards; to find the area.

Length AB - - - 242
Breadth EF - - - 160

14520
242

The product is - - 38720 the area required.

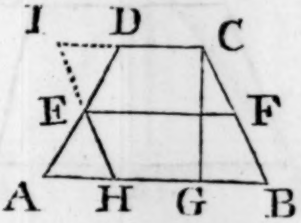
REMARK.

If the area of a field in yards is divided by 4840 *(the number of ſquare yards in one acre) the quotient will give the number of acres in that field. In this example there are eight acres.*

A

A Theorem.

347. *Every quadrangle having two parallel ſides, is equal to a rectangle contained by half their ſum, and a perpendicular between them.*



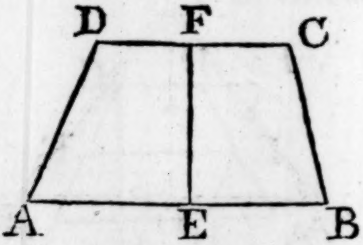
Let ABCD be a quadrangle having the ſide DC parallel to the baſe AB, biſect AD in E, draw EF parallel to AB or DC, and let fall the perpendicular CG; then EF = half $\overline{AB+DC}$, and ABCD = EF×CG.

For thro' the point E draw HI parallel to BC, and produce CD to I.

Now the angles AHE, EAH are equal to DIE, EDI. 82
And the correſponding ſides AE, ED are equal. con.
Th. AH = ID, and the triangle AHE = EID - 77
Th. ABCD = HBCI - - - - - 49
But HB×CG = HBCI - - - - - 94
Th. ABCD = HB×CG - - - - 48
Again, EF, HB, IC are equal - - - 91
Th. 2EF = HB + ID + DC - - - - 49
Or 2EF = HB + AH + DC = AB + DC - - 48
Th. EF = half $\overline{AB+DC}$ - - - - 51
And ABCD = EF × CG - - - - 56

Problem 5.

348. *To find the area of a quadrangle* ABCD, *having two parallel ſides* AB, DC.



RULE.

Multiply half the ſum of the parallel ſides by the perpendicular between them; and the product will give the area.

EXAMPLE.

Let the parallel ſides AB, DC, and the perpendicular between them EF be 955, 637, and 630 links of Gunter's chain reſpectively; to find the area of the figure.

Now AB - - = 955
- - - DC - - = 637

Their ſum - - = 1592

The half ſum - - = 796
The perpendicular EF = 630 ×

23880
4776

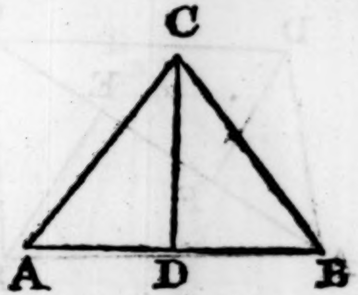
The product - - = 501480. the area in links.

REMARK.

If the area of a field in links is divided by 100000 *(the number of ſquare links in one acre) the quotient will expreſs the number of acres in that field—Thus in the example above, the field contains five acres.*

Problem 6.

349. *To find the area of a triangle* ABC.



RULES.

1. *Multiply the baſe by half the altitude, or the altitude by half the baſe, and the product gives the area.* Or

2. *Half the product of the baſe and altitude will give the area.*

Becauſe a triangle is equal to half a parallelogram of the ſame baſe and altitude - - - 98

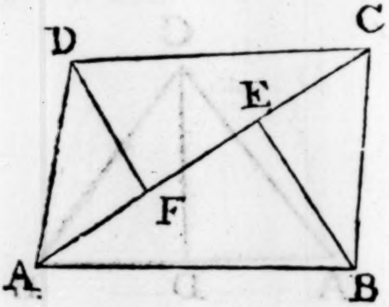
EXAMPLE.

Let the baſe AB be 97, and the altitude or perpendicular CD be 68, to find the area.

Now AB =	97	- - -	AB = 97	ſecondly.
Half CD =	34	- - -	CD = 68	
	388		776	
	291		582	
The area =	3298	- - -	= half 6596 = 3298.	

Problem 7.

350. *To find the area of any quadrangle* ABCD.



RULE.

Meaſure a diagonal line AC, *and the perpendiculars* BE, DF *falling upon it from the oppoſite angles; multiply the ſum of theſe perpendiculars by half the diagonal, and the product will give the area; or multiply the ſum of the perpendiculars by the diagonal, and half the product will give the area.*

This rule ariſes from the preceding, and is only determining two triangles at one operation.

EXAMPLE.

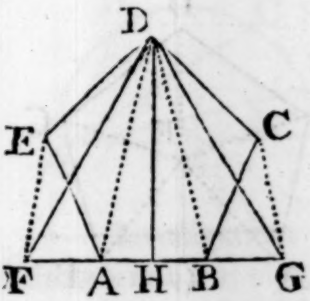
Let AC, BE, DF be 18, 8, and 7 of Gunter's chain reſpectively, to find the area.

Now $8 + 7 = 15$ the ſum of the perpendiculars.

Th. $15 \times 9 = 135$, the area in chains, which divided by 10 (the number of ſquare chains in one acre) gives 13,5 acres; namely, thirteen acres and a half.

Problem 8.

351. *To find the area of any ſtrait lined figure.*



RULES.

1. *Divide the figure into triangles, find the area of each triangle, by problem* 6, *and their ſum will be the content.* Or

2. *Make a triangle equal to the given figure, by article* 312, *and find the area of this equal triangle.*

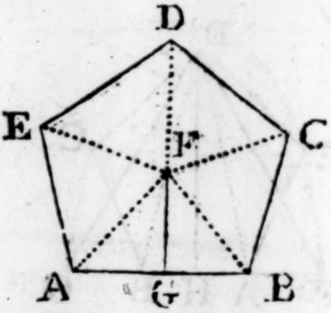
EXAMPLE.

Let ABCDE be the given figure, and FGD be a triangle made equal to it, whoſe baſe FG meaſures 1244 links, and perpendicular DH meaſures 1120 links.

Then FG - - - - = 1240
Half DH - - - - = 560
74400
6200
The area of FGD, or ABCDE - = 694400 links.
Which is - - - - 6,94400 acres.

Problem 9.

352. *To find the area of any regular polygon.*



RULE.

Let fall a perpendicular from the center of the figure to one of its ſides; then multiply together the perpendicular, the ſide of the figure, and the number of its ſides; and half the product will expreſs the area.

For if lines be drawn from every angular point to the center of the figure, the polygon will be divided into the ſame number of equal triangles, as it has ſides.

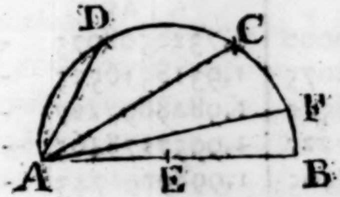
EXAMPLE.

What is the area of a regular pentagon ABCDE, whoſe ſide AB is 250, and perpendicular FG is 172?

Now 172×250×5 = 215000 the product of the three given quantities, and its half is 107500, the content required.

Problem 10.

353. *To find the circumference of a circle whoſe diameter is 2.*



Inveſtigation.

In the ſemicircle ABCD, apply the line AD equal to the radius AE, and draw AC biſecting the angle BAD.

Now AD is the ſide of a regular hexagon (335) and the arches BC, CD are equal (148) therefore each is $\frac{1}{6}$ of the whole circumference, and the line AC is given by 220. By the ſame theorem, a ſeries of biſecting lines may be found approaching to the diameter AB, and every intercepted arch (BC) will be a known part of the circumference; and having thus determined an arch BF ſufficiently minute for our purpoſe, by joining BF, the triangle AFB is right angled at F (153) and the line BF is given (100.) Laſtly, BF being the ſide of a regular inſcribed figure, whoſe number of ſides is given, the circuit of that polygon is given, and conſequently the circumference of the circle (being greater than the circuit of any inſcribed polygon) is nearly found, but to determine it exactly is impoſſible.

Opera-

Operation.

N°	AC^2	AC	Ar. BF:
1	3,0000000000	1,7320508075 -	$\frac{1}{6}$
2	3,7320508075	1,9318516525 -	$\frac{1}{12}$
3	3,9318516525	1,9828897227 -	$\frac{1}{24}$
4	3,9828897227	1,9957178465 -	$\frac{1}{48}$
5	3,9957178465	1,9989291743 -	$\frac{1}{96}$
6	3,9989291743	1,9997322757 -	$\frac{1}{192}$
7	3,9997322757	1,9999330678 -	$\frac{1}{38}$
8	3,9999330678	$BF^2 = 0,0000669322$	$\frac{1}{768}$

Therefore BF = 0,00818121, and 0,00818121 × 768 = 6,28317, the circuit of an inſcribed polygon, having 768 ſides.

Again, ſince the inſcribed polygon of 768 ſides is determined, the circuit of a circumſcribing polygon ſimilar to it is eaſily found to be 6,28322, (the circuits and perpendiculars from the center to their ſides being proportional) and conſequently the circumference of the circle is nearly 6,2832, which is a mean between them.

Corollaries.

Corollaries.

354. *If the diameter of a circle is* 1, *the circumference is* 3,1416.

355. *If the diameter of a circle is multiplied by* 3, 1416 *the product will give the circumference.*

356. *If the radius of a circle is multiplied by* 6,2832, *the product will give the circumference.*

357. *As* 7 : 22 :: *diameter* : *circumference.*

358. *As* 7 : 44 :: *radius* : *circumference.*

359. *If the circumference of a circle is divided by* 3, 1416, *the quotient will give the diameter.*

360. *If the circumference of a circle is divided by* 6, 2832, *the quotient will give the radius, or ſemidiameter.*

Becauſe the circumſerences of circles are proportional to their diameters, or ſemidiameters - - 225

Remark.

Since the circumference of the circle is only determined nearly, and not accurately; ſo the corollaries above are nearly true only, but not exactly ſo.

Problem

Problem 11.

361. *To find the area of a circle, whoſe diameter and circumference are given.*

RULES.

1. *Multiply half the circumference, by half the diameter, and the product will expreſs the area.* Or,

2. *Multiply the circumference by the diameter, and a fourth part of the product will expreſs the area.*

Becauſe a circle is equal to a triangle, whoſe baſe is equal to the circumference, and altitude is equal to the ſemidiameter. - - - - - 227

EXAMPLE.

What is the area of a circle, whoſe diameter is 2, and circumference is - - 6,2832?

Now half the circumf. = 3,1416
- - - half the diameter = - - - 1

Th. the area - - - = 3,1416 by rule firſt.

EXAMPLE 2.

What is the area of a circle, whoſe diameter is 1, and circumference is - - 3,1416?

Multiply by the diameter - - 1
The product is - - = 3,1416

Th. the area - - = 0,7854 by rule ſecond.

Problem 12.

362. *The diameter, or ſemidiameter of a circle being given, to find the area of that circle.*

RULES.

1. *Multiply the ſquare of the diameter by* 0,7854, *and the product will give the area.* Or

2. *Multiply the ſquare of the ſemidiameter by* 3,1416, *and the product will give the area.*

Becauſe 0,7854 and 3,1416 are the areas of circles, whoſe diameters are 1 and 2 (361) and the areas of circles are proportional to the ſquares of their diameters, or ſemidiameters - - - - - 225

Again, 1 : 0,7854 :: 14 : 11, nearly,
And 1 : 3,1416 :: 7 : 22, nearly.

Hence the following RULES.

3. *As* 14 *to* 11, *ſo is the ſquare of the diameter to the area of the circle* - - - - - 173

4. *As* 7 *to* 22, *ſo is the ſquare of the ſemidiameter to the area of the circle* - - - - - 173

REMARK.

The area of a circle cannot be found exactly, becauſe the diameter and circumference, are not (both of them) to be accurately expreſſed by numbers - - - 353

EXAMPLE.

What is the area of a circle whoſe diameter is 12?

Now $12 \times 12 = 144$, the ſquare of the diameter, and $0,7854 \times 144 = 113,0976$, the area required, by rule the firſt.

Secondly, $6 \times 6 = 36$, the ſquare of the ſemidiameter; and $3,1416 \times 36 = 113,0976$, the area required, by rule the ſecond.

Thirdly, $14 : 11 :: 12 \times 12$: area of the circle, which therefore $= \frac{12 \times 12 \times 11}{14} = \frac{1484}{14} = 113,1$, by rule the third.

Laſtly, $7 : 22 :: 6 \times 6$: area of the circle, which conſequently $= \frac{6 \times 6 \times 22}{7} = \frac{792}{7} = 113,1$, by rule the fourth.

Problem 13.

363. *The circumference of a circle being given, to find the area.*

RULES.

1. *Find the ſemidiameter by the* 360, *and then find the area by the* 361. Or

2. *Multiply the ſquare of the circumference by* 0,079577*, and the product will give the area.*

For the ſquares of the circumferences, are as the ſquares of the diameters (207) therefore the areas are as the ſquares of the circumferences (173) and 0,079577 is the area of a circle whoſe circumference is 1.

EXAMPLE.

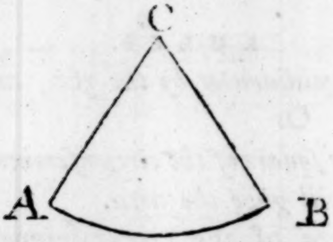
What is the area of a circle whoſe circumference is 24?

Now the ſemidiameter is 3,8197, and 3,8197 × 12 = 45,8364 the area required, by rule the firſt. Or

Secondly, the ſquare of the circumference 24 × 24 = 576, and 0,079577 × 576 produces 45,836352 for the area required.

Problem 14.

364. *To find the area of a ſector of a circle* ABC.



RULE.

Multiply the length of the arch by the radius of the circle, and half the product will give the area. Or multiply either of them by half the other, and the product will expreſs the area.

For a ſector of a circle is equal to a triangle, whoſe baſe is equal to the length of the arch, and altitude is equal to the radius of the ſector - - - 227

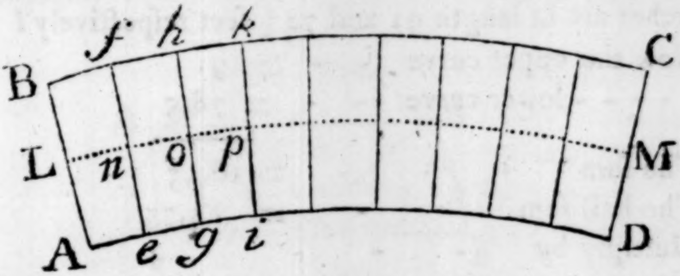
EXAMPLE.

Let the radius CA be 55, and the length of the arch AB be 59.

Now the arch AB - - = 59
The radius CA - - = 55
295
295
- - - the product - - = 3245
Half the product - - = 1622,5 the area of ABC.

Problem 15.

365. *To find the area of a ſegment of a ſector* ABCD, *or the front of an arch built with ſtones of equal length.*



RULE.

Multiply half the ſum of the bounding arches AD, BC; *by their diſtance* AB, *and the product will give the area.*

For let the ſegment be divided into equal parts indefinitely ſmall, by ſtrait lines e f, g h, i k, &c. drawn from the common center of the arches AD, BC; and about the ſaid center deſcribe the arch LM biſecting AB or DC, and cutting e f in n.

Now the parts A f, e h, g k, &c. repreſenting the fronts of arch ſtones indefinitely thin, they may be taken for quadrangles, having their upper and lower ſides parallels, and being all equal to one another, each is equal to L n × AB (347); therefore the whole ſegment ABCD = LM × AB (106) and L n being half the ſum of B f and A e, LM muſt be half the ſum of BC and AD; which gives the rule.

EXAMPLE.

What is the area of the front of an arch, built with ſtones of 4 feet long, whoſe upper and lower bounding arches are in length 91 and 78 ½ feet reſpectively?

Now the upper curve - - = 91
- - - - - lower curve - - = 78,5

The ſum - - - = 169,5
The half ſum - - = 84,75
Multiply by - - - 4

The area required - - 339 ſquare feet

EXAMPLE 2.

What is the area contained between two concentric ſemicircles, whoſe diameters are 24 and 16?

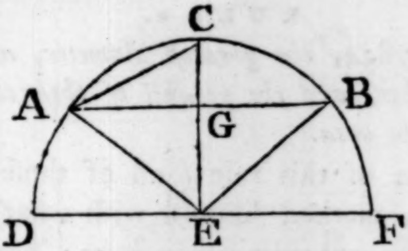
Now the greater ſemicircle - - = 37,7 } by 357
- - - the leſſer ſemicircle - - = 25,1 }

Their ſum - - - - - = 62,8

The half ſum - - - - = 31,4
Multiply by - - - - - 4 the diſtance between the circles, and the product 125,6 is the area required.

Problem 16.

366. *To find the area of a ſegment of a circle* ABC, *whoſe center is* E.



RULES.

1. *Find the area of the triangle* ABE, *and of the ſector* ACBE, *by articles* 349, 364; *and their difference is the area of the ſegment* ABC. Or

2. * *To ſix times the baſe, add eight times the chord of half the arch, multiply the ſum by the altitude, divide the product by* 15, *and the quotient will nearly give the area.*

EXAMPLE for RULE 2*.

Let the baſe AB be 8, and the altitude CG be 3. Now $AC^2 = AG^2 + GC^2 = 16 + 9 = 25$; th. $AC = 5$. Th. $8 \times 6 + 5 \times 8 = 48 + 40 = 88$, the ſum to be multiplied; th. $88 \times 3 = 264$ the product; which, divided by 15, gives 17,6 the area of the ſegment ABC, nearly.

Problem 17.

367. *To find the area of an ellipſis, or oval.*

RULE *.

Multiply 0,7854, *the greateſt diameter, and the leaſt diameter together, and the product of theſe three numbers will expreſs the area.*

The reaſon of this rule (and of thoſe in articles 366 and 385, marked likewiſe with a ſtar) cannot be given from the preceding principles; but ſince they may prove very uſeful to artificers, it is hoped the more ſkilful reader will excuſe their being inſerted.

EXAMPLE.

What is the area of an ellipſis, whoſe greateſt diameter is 24, and leaſt diameter is 18?

Now the conſtant number is - - - 0,7854
- - - the greateſt diameter is - - 24 ×

31416
15708

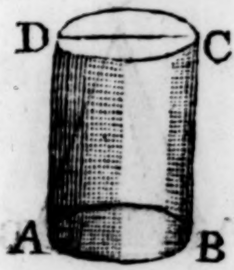
The firſt product is - - - 18,8496
The leaſt diameter is - - - 18 ×

1507968
188496

The area is - - - - - 339,2928

Problem 18.

368. *To find the convex ſurface of a right cylinder* ABCD.



RULE.

Multiply the circumference of the baſe by the altitude of the cylinder, and the product will give the convex ſurface.

For conceiving the convex ſurface cut parallel to the axis, and then ſpread ſmooth upon a plain; it will evidently fall into a rectangular figure, and conſequently muſt be determined after the ſame manner. 345

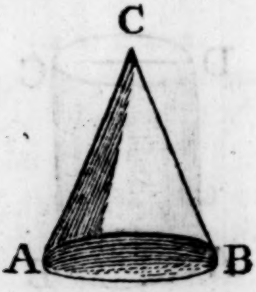
EXAMPLE.

What is the convex ſurface of a right cylinder, whoſe circumference is 9½ feet, and length is 4½ feet?

	1. Decimally.	2. By croſs multiplication
		f. : *i.*
Circumference -	= 9,5	9 : 6
Length - - -	4,5	4 : 6
	475	4 : 9 : 0
	380	38 : 0
Convex ſurface	= 42,75	= 42 : 9 : 0

Problem 19.

369. *To find the convex ſurface of a right cone* ABC.



RULE.

Multiply the circumference of the baſe by the ſlant ſide, and half the product will give the area.

For conceiving the ſurface cut in a ſtrait line from the vertex to the baſe, and then ſpread ſmooth upon a plain, it will evidently fall into the ſector of a circle; and conſequently, muſt be determined after the ſame manner - - - - - - - 364

EXAMPLE.

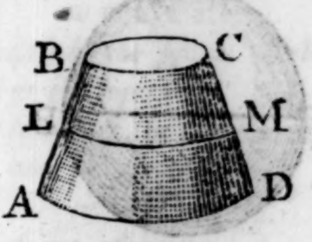
What is the convex ſurface of a right cone, whoſe baſe is 64 feet in circumference, and ſlant ſide AC is 28 feet in length?

```
Now the circumference  -  -  =  64
Slant ſide AC   -   -   -    =  28 ×
                               ———
                               512
                              128
                              ————
The product    -   -   -   =  1792
The convex ſurface  -  -   =   896 ſquare feet.
                               ———
```

Problem

Problem 20.

370. *To find the convex ſurface of the fruſtum of a right cone* ABCD, *made by a ſection parallel to the baſe.*



RULE.

Multiply half the ſum of the circumferences of the ends by the ſlant ſide; and the product will give the convex ſurface.

For conceiving the convex ſurface cut in the ſtrait line AB, and then ſpread ſmooth upon a plain, it will evidently fall into the ſegment of a ſector; whoſe bounding arches are equal to the circumferences of the ends, and whoſe ſides are equal to the ſlant ſide of the fruſtum AB or DC; likewiſe the circumference of a circle LM round the middle of the fruſtum, will fall into an arch LM biſecting the ſides of the ſegment: wherefore the convex ſurface is truly expreſſed by LM × AB. - - - - - - - 365

EXAMPLE.

Let the circumferences of the ends be 32, and 8 feet, and the length of the ſlant ſide AB be 7 feet; to find the convex ſurface. Now half $\overline{32 + 8} \times 7 = 20 \times 7 = 140$, the content required.

Problem 21.

371. *The diameter of a globe being given; to find the ſuperficies.*

RULE.

Find the circumference of a great circle upon the globe by article 356, multiply the circumference by the diameter, and the product will expreſs the ſuperficies.

Becauſe the ſurface of a globe is four times the area of its great circle (283) and the product of the circumference, by the diameter, is likewiſe four times the area of the ſame circle - - - - - 362

EXAMPLE.

What is the ſuperficies of a globe, whoſe diameter AB is 1?

Now the circumf. of a great circle = 3,1416. 355
- - - the diameter - - - = 1 ×

The ſuperficies of the globe - - = 3,1416

Problem 21 a.

372. *The diameter, or ſemidiameter of a globe being given, to find the ſuperficies.*

RULES.

1. *Multiply* 3,1416 *by the ſquare of the diameter, and the product will give the ſuperficies.* Or,

2. *Multiply the ſquare of the ſemidiameter by* 88, *divide the product by* 7, *and the quotient will give the ſuperficies.*

Becauſe the ſurfaces of globes, are proportional to the ſquares of their diameters, or ſemidiameters (290) and 3,1416, is the ſuperficies of a globe whoſe diameter is 1 - - - - - - 371

EXAMPLE.

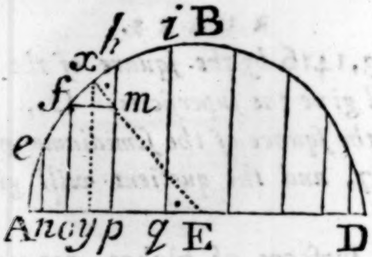
What is the ſuperficies of a globe whoſe diameter is 8 feet?

First method.		Second method.	
Conſtant No.	3,1416	Square of ſemidiam.	16
Square of diameter	64	Conſtant number	=88
	125664		128
	188496		128
Superficies	201,0624	Square feet.	7)1408(201

Problem

Problem 22.

373. *To find the ſuperficies of a ſegment of a globe, made by the ſection of a plain.*



RULE.

Multiply the circumference of the globe by the height of the ſegment, and the product will give the ſuperficies.

For let a ſemicircle ABD be divided into parts a e, e f, f h, &c. indefinitely ſmall, ſo as to be taken for ſtrait lines, biſect f h in x, draw e n, f o, x y, &c. perpendicular to AD, and f m parallel to AD, join the point x to the center E, and ſuppoſe the ſemicircle to turn about the axis AD and generate a globe.

Now E x f is a right angle (126) th. the angles x f m, E x y are equal, and the triangles h f m, E x y are equiangular; th. h f : f m :: E x : x y; but E x : x y :: the circumference of the globe c : the circumference of a circle, whoſe radius is y x (225) th. h f : f m :: c : circumference x (173) th. circumference x × h f = c × f m, or c × o p; but the ring, or zone deſcribed by f h, being the convex ſurface of the fruſtum of a right cone = circumference x × f h (370) th. the zone deſcribed by f h = c × o p (48). By the ſame reaſon, each zone deſcribed upon the globe, is equal to the product of the circumference c by the intercepted part of the axis, and conſequently a ſuperficies deſcribed by any arch A f, is equal to c × A o.

Corollary.

Corollary.

374. *If the parts of the diameter a i, i g, g n, &c. are equal, the zones deſcribed by the correſponding arches a e, e f, f h, &c. are likewiſe equal.*

Remark.

The ſuperficies of a globe is expreſſed by the product of its circumference and diameter, as before determined in article 371.

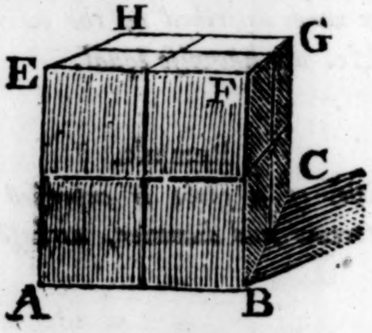
EXAMPLE.

What is the ſuperficies of a ſegment 9 feet high, cut from a globe of 42 feet diameter?

The conſtant number - - = 3,1416
The diameter of the globe - = 42
62832
125664
The circumference - - = 131,9472, by article 355.
The height of the ſegment - - - 9 ×
The ſuperficies required = 1187,5248

Problem 23.

375. *To find the ſolidity or content of a cube* ABCH.



RULE.

Multiply the ſquare of a ſide by the ſide, and the product will expreſs the content.

For if a cube ABCH be compoſed or built up with leſſer cubes, the number of them placed upon the baſe is equal to the number of little ſquares in the baſe, and that number is expreſſed by AB × AB, the ſquare of the ſide (344). Again, the number of courſes is equal to the number of parts or meaſures in the ſide AE or AB; and conſequently the number of leſſer cubes contained in the greater, is expreſſed by AB×AB×AB, which gives the rule above.

EXAMPLE.

What is the content of a cube whoſe ſide is two feet.

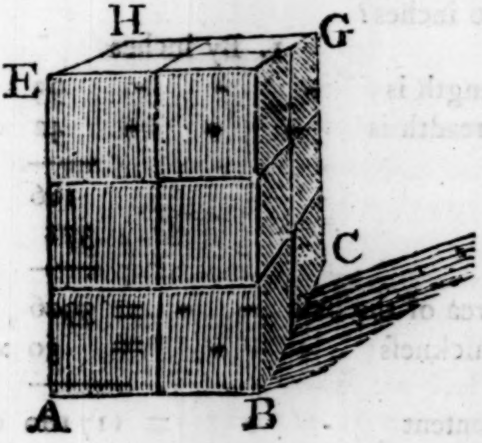
Now the ſide AB or AE - - = 2
- - - - - - = 2

The ſquare of a ſide - - = 4 = $AB \times AB$, or AB^2
Again, the ſide AB - - = 2

The content required - - = 8 = $AB^2 \times AB$.

Problem 24.

376. *To find the content of a priſm* ABCDEFGH.



RULE.

Multiply the area of the baſe by the height or altitude, and the product will expreſs the content.

For if a priſm is erected upon a ſquare baſe, and compoſed or built up with equal cubes, the number placed upon the baſe muſt be equal to the number of little ſquares in the baſe, and the number of courſes will be equal to the number of parts or meaſures in the altitude, wherefore the content is truly expreſſed as above in a ſquare priſm; and all priſms of equal baſe and altitude being equal (268) the rule is true univerſally.

EXAMPLE.

Let the area of the baſe ABCD be 4 feet, and the height AE be 3 feet, to find the content.

Now $4 \times 3 = 12$, the content required in cubic feet.

EXAMPLE 2.

What is the content of a block of marble, in length 7 feet 9 inches, breadth 3 feet 6 inches, and thickneſs 2 feet 6 inches?

1. By inches

The length is - - - 93
The breadth is - - - 42 ×

186
372

The area of the baſe - - = 3906
The thickneſs - - = 30 ×

The content - - = 117180 cubic inches, which divided by 1728 (the number of cubic inches in a cubic foot) gives 67,8 cubic feet, the content required.

2. By croſs multiplication.

f. i.
Again, the length is - 7 : 9
- - - - the breadth is - 3 : 6 ×

3 : 10 : 6
23 : 3

Area of the baſe - - 27 : 1 : 6
The thickneſs - - 2 : 6 ×

13 : 6 : 9 : 0
54 : 3 : 0

The content is - - 67 : 9 : 9 : 0 cubic feet.

Problem

EXAMPLE 3.

What is the content of a ſquare ciſtern, whoſe length, breadth, and depth are 54, 27, and 33 inches reſpectively?

$$\begin{array}{lrl}
\text{Now the length - - =} & 54 & \\
\text{- - - - - breadth - - =} & 27 & \times \\
\hline
 & 378 & \\
 & 108 & \\
\hline
\text{The product - - =} & 1458 & \text{the area of the bottom} \\
\text{The depth - - =} & 33 & \times \\
\hline
 & 4374 & \\
 & 4374 & \\
\hline
\text{The content - - =} & 48114 & \text{cubic inches, which,}
\end{array}$$

divided (by 282, the number of cubic inches in a gallon) gives 170 gallons, the content required.

EXAMPLE 4.

What is the content of a canal 20 feet broad at the top, 18 feet broad at bottom, 5 feet deep, and 300 feet long?

This ſolid being a priſm, whoſe ends or baſes are quadrangles having two parallel ſides, the area of the baſe (by article 348) = half $\overline{20+16}\times 5 = 18\times 5 = 90$, which, multiplied by 300, the length or height gives 90 × 300, or 27000 cubic feet, the content required.

Problem 25.

377. *To find the content of a wall, in ſquare rods of a brick and half thick.*

RULE.

Multiply the area of the wall in feet, by the number of half bricks in thickneſs, divide the product by 816, *and the quotient will expreſs the content required.*

Becauſe $15\frac{1}{2} \times 15\frac{1}{2} \times 3$, or $272\frac{1}{4} \times 3$, or $816\frac{3}{4}$, expreſſes the content of a wall that is a ſquare rod in area, and three half bricks in thickneſs (376); which ſolid is called a ſquare rod of work; the fraction $\frac{3}{4}$ being rejected in favour of the workmen.

EXAMPLE.

How many ſquare rods of work is there in a wall $187\frac{1}{2}$ feet long, 7 feet high, and $2\frac{1}{2}$ bricks thick?

Now the length of the wall is 187,5
- - - - - height - - - 7 ×

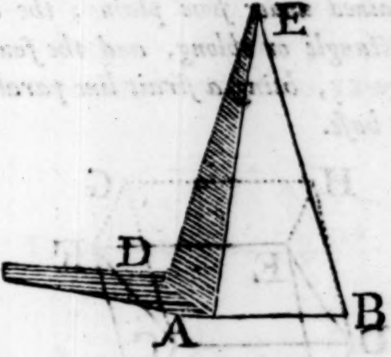
The area of the wall is - - 1312,5
Multiply by - - - - 5 in thickneſs

The number to be divided is 6562,5, which, divided by 816 gives 8 rods, the content required.

Problem

Problem 26.

378. *To find the content of a pyramid* ABCDE.



RULE.

Multiply the area of the baſe, by a third part of the altitude, and the product will give the content.

Becauſe a pyramid is a third part of a priſm, having the ſame baſe and altitude - - - - 275

EXAMPLE.

What is the content of a pyramid 300 feet high, erected upon a baſe 24 feet ſquare?

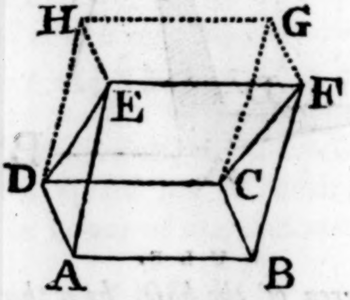
Now the ſide of the baſe is - - 24
× 24
96
48

The area of the baſe - - - = 576
One third of the height - - = 100

The content - - - = 57600 cubic feet

Problem 27.

379. *To find the content of a wedge* ABCDEF, *which is a ſolid contained under five plains; the back or baſe* ABCD, *is a rectangle or oblong, and the four ſides terminate in the edge* EF, *being a ſtrait line parallel and equal to a ſide of the baſe.*



RULE.

Multiply the area of the baſe, by half the altitude of the edge, and the product will give the content.

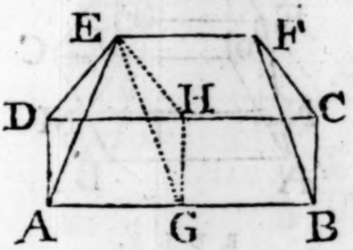
For this ſolid is likewiſe a triangular priſm, whoſe baſes are ADE, BCF; and if plains be drawn through EF, DC parallel to AC, AF, the ſolid ABCDEFGH formed thereby will alſo be a priſm; and either the rectangle AC, or parallelogram AH, may be taken for its baſe (234). Th. the priſm ADEBCF = HEDGFC (268). Th. ADEBCF, or ABCDEF is half ABCDEFGH, which gives the rule above.

EXAMPLE.

What is the content of a wedge whoſe baſe meaſures 36 feet by 20, and whoſe height is 12 feet?
Now 36 × 20 = 720, the area of the baſe. Th. 720 × 6 = 4320, the content required.

Problem 28.

380. *To find the content of a pavilion roof* ABCDEF, *which is a ſolid contained under five plains; the baſe is a rectangle or oblong, and the four ſides terminate in a ridge* (EF) *parallel to a ſide of the baſe, but unequal to it.*



RULE.

To the length of the ridge, add twice the ſide of the baſe which is parallel to it. Multiply the ſum by the other ſide of the baſe, and the product which ariſes by a ſixth part of the altitude, and the ſecond product will give the content.

For ſuppoſing the ſection EGH, made parallel to the plain FBC, the roof is then divided into the pyramid AGHDE, and the wedge GBCHEF; now calling the altitude a, and finding the contents of thoſe parts according to the preceding rules, their ſum (properly ordered) will be expreſſed by $\overline{EF+2AB}\times BC\times\frac{a}{6}$, which gives the rule above.

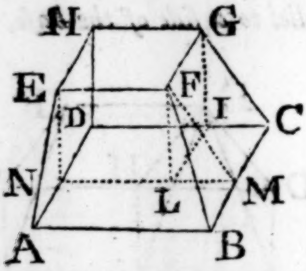
EXAMPLE.

What is the content of a pavilion roof, whoſe baſe is 36 feet by 20, ridge parallel to the greater ſide is 16, and altitude is 12 feet?

Now 16 + 72 = 88, the ſum to be multiplied; th. 88 × 20 × 2 = 3520 cubic feet, the content required.

Problem 29.

381. *To find the content of the fruſtum of a ſquare pyramid* ABCDEFGH, *made by a ſection paralel to the baſe.*



RULE.

To the areas of the ends, add the product of their ſides, multiply the ſum by a third part of the altitude, and the product will give the content.

For let the ſections FGIL, EFMN be made parallel to the plains AH, HC, and the fruſtum will be divided into the priſm NLIDEFGH, the wedge MCILFG, and the pavilion roof ABMNEF; now calling the altitude a, and finding the contents of thoſe parts according to the preceding rules, their ſum (properly ordered) will be expreſſed by $\overline{AB^2 + EF^2 + AB \times EF} \times \frac{a}{6}$, which gives the rule above.

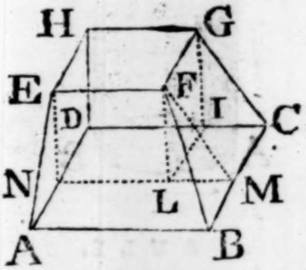
EXAMPLE.

What is the content of the fruſtum of a pyramid 60 feet high, whoſe ends are 16 and 13 feet ſquare?

Now 16×16 = 256, 13 × 13 = 169, and 16 × 13 = 208, which are the areas of the ends, and the product of their ſides; th. 256 + 169 + 208 = 633, the ſum to be multiplied; th. 633 × 20 = 12660, the content required.

Problem 30.

382. *To find the content of a priſmoid* ABCDEFGH, *being a ſolid contained under ſix plains; the baſes or ends are parallel rectangles or oblongs, and the four ſides are quadrangles.*



RULE.

To the areas of the ends, add the product of the ſums of their lengths and breadths; multiply this ſum by a ſixth part of the altitude, and the product will give the content.

For let the ſections FGIL, EFMN be made parallel to the plains AH, HC, and the priſmoid will be divided into the priſm NLIDEFGH, the wedge MCILFG, and the pavilion roof ABMNEF; now calling the altitude a, and finding the contents of theſe parts by the preceding rules, their ſum (properly ordered) will be expreſſed by $\frac{AB \times BC + EF \times FG + \overline{AB+EF} \times \overline{BC+FG}}{1} \times \frac{a}{6}$, which gives the rule above.

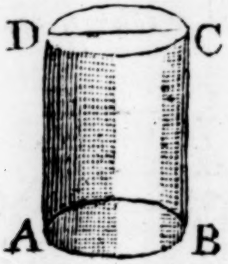
EXAMPLE.

What is the content of a canal 304 feet by 20 at top, 300 feet by 16 at bottom, and 5 feet deep?

Now 304 × 20 = 6080, 300 × 16 = 4800, and 604×36 = 21744, which are the numbers to be added; th. 6080 + 4800 + 21744 = 32644, the ſum to be multiplied; th. $32624 \times \frac{5}{6} = 27186$ the content required.

Problem 31.

383. *To find the content of a cylinder* ABCD.



RULE.

Multiply the area of the baſe by the altitude of the cylinder, and the product will expreſs the content.

For all priſms and cylinders of equal baſe and altitude are equal (268); and therefore muſt be determined by the products of their baſes and altitudes. 376

EXAMPLE.

What is the content of a cylinder 3 feet diameter, and 6 feet high?

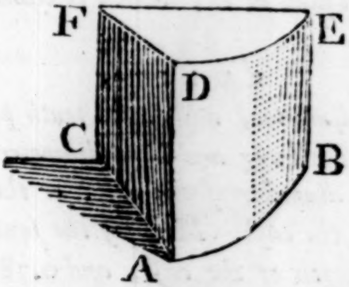
Now 3 × 3 = 9, the ſquare of the diameter.

Th. 14 : 11 :: 9 : 7,07, the area of the baſe, by rule 3, article 362.

Th. 7,07 × 6 = 42,42 the content required.

Problem 32.

384. *To find the content of a triangular ceſtern, whoſe bottom is the ſector of a circle.*



RULE.

Multiply the area of the bottom in inches, by the depth in inches; divide the product by 282, and the quotient will be the content in gallons.

For this ſolid is evidently a portion of a cylinder, and conſequently muſt be determined by a ſimilar rule.

EXAMPLE.

What is the content of a ceſtern whoſe bottom is a quarter of a circle 21 inches in ſemidiameter, and whoſe depth is 42 inches?

Now by rule 4, article 362, the area of a circle 21 inches in ſemidiameter is 1386, its fourth part 346,5 is the area of the bottom of the ceſtern, and 346,5 × 42 = 14553 the content in cubic inches, which, divided by 282 gives 51,6 gallons, the content required.

Problem 33.

385. *The bung diameter, head diameter, and length of a caſk (within ſide) being given; to find the content of a cylinder nearly equal to it; which is called gaughing the caſk.*

RULE *.

To the head diameter, add ſeven tenth parts of the difference between the bung and head diameters, and the ſum will be a mean diameter of the caſk, or the diameter of a cylinder equal to the caſk. Multiply the ſquare of the mean diameter, the length of the caſk, and 0,78 *together; and the product will be the content, nearly.*

Note. The number ($\frac{7}{10}$) uſed in finding a mean diameter, is thought the beſt adapted to a general rule.

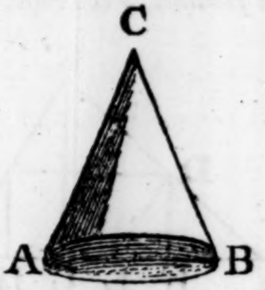
EXAMPLE.

What is the content of a caſk, whoſe bung diameter, head diameter, and length, are 32,26 and 40 inches, (within ſide) reſpectively?

Now $32 - 26 = 6$, the difference of diameters, and $6 \times 0{,}7 = 4{,}2$ the number to be added; th. $26 + 4{,}2 = 30{,}2$ the mean diameter, whoſe ſquare is $30{,}2 \times 30{,}2$ or 912,04; th. $912{,}04 \times 40 \times 0{,}78 = 2845$, the content in cubic inches, which, divided ſeperately by 282, and 231, will give 101 ale, and 123 wine gallons, the contents required.

Problem 34.

386. *To find the content of a cone* ABC.



RULE.

Multiply the area of the baſe by a third part of the altitude, and the product will give the content.

Becauſe a cone is a third part of a cylinder, having the ſame baſe and altitude. - - - 276

EXAMPLE.

What is the content of a cone whoſe baſe is 3 feet diameter, and altitude is 6 feet?

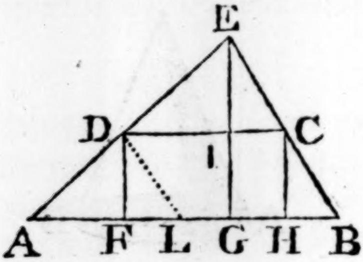
Now 3×3=9 the ſquare of the diameter of the baſe.

And 14 : 11 :: 9 : 7,07, the area of the baſe, by rule 3, article 362.

Th. 7,07 × 2 = 14,14, the content required.

Problem 35.

387. *To find the content of the fruſtum of a cone* ABCD, *made by a ſection parallel to the baſe.*



RULES.

To the ſquares of the $\left\{\begin{matrix}\textit{diameters}\\ \textit{circumferences}\end{matrix}\right\}$ *of the ends add their product; multiply the ſum by the altitude of the fruſtum, and the product which ariſes by* $\left\{\begin{matrix}0{,}2618\\ 0{,}0265\end{matrix}\right\}$ *and this laſt product will give the content.*

For ſuppoſe E the vertex of the compleat cone, and the triangle ABE a ſection thro' the axis; draw DL parallel to EB, and let fall the perpendiculars DF, EG, CH.

Now LB is equal to DC (91) and the triangles ALD, ABE, DCE are equiangular (83); th. AL : DF :: AB : EG :: DC : EI (201); whence the altitudes EG, EI are determined; now calling the diameters of the ends D, d, their circumferences C, c; and finding the contents of the cones ABE, DCE, by article 386; their difference will produce the following expreſſions, namely,

1. $ABCD = \overline{D^2 + d^2 + D \times d} \times DF \times 0{,}2618.$

2. $ABCD = \overline{C^2 + c^2 + C \times c} \times DF \times 0{,}0265,$ which give the rules above.

EXAMPLE 1.

What is the content of the fruſtum of a cone 60 feet high, the diameters of its ends being 20 and 3 feet?

Now 20 × 20 = 400, 3 × 3 = 9, and 20 × 3 = 60, which are the ſquares of the diameters and their product. Th. 400 + 9 + 60 = 469, the ſum to be multiplied.

The altitude - - = 60

Firſt product - - 28140
× 0,2618

215120
28140
168840
56280

Content - - = 7366,0520

EXAMPLE 2.

What is the content of a conical fruſtum, the circumferences of whoſe baſes are 66 and 56 feet, and whoſe height is 4 feet?

Here 66 × 66 = 4356, 56 × 56 = 3136, and 66×56 = 3696; which are the ſquares of the circumferences, and their product. Th. 4356 + 3136+3696 = 11188 the ſum to be multiplied by - - - 4

The firſt product is - - - - - 44752
× 0,0265

223760
268512
89504

The content required - - - = 1185,9280

Problem 36.

To meaſure TIMBER.

388. A ſquare piece of timber equally thick at both ends is a priſm, a round piece equally thick at both ends is a cylinder; a ſquare piece that tapers regularly is the fruſtum of a pyramid, and a round piece that tapers regularly is the fruſtum of a cone; and the contents of theſe ſolids may be exactly computed by their reſpective rules.

But becauſe the menſuration of tapering timber by the exact rules is troubleſome, an approximation has taken place, and the contents of ſuch trees are generally computed by the following

RULE.

Multiply the ſquare of the girt in inches by the length in feet, divide the product by 144, *and the quotient will give the content in feet.*

Remarks.

1. *The girt of a piece of timber is a fourth part of its compaſs or circumference at the middle.*

2. *Trees of irregular growth muſt be meaſured in parts or pieces, as above directed.*

3. *Allowance muſt be made for the thickneſs of bark, if on the tree.*

EXAMPLE 1.

What is the content of a tree whoſe girt is 16 inches, and length is 30 feet?

Now 16 × 16 - - = 256 the ſquare of the girt
Multiply by - - - 30 the length

The product is - - 7680, which, divided by 144 gives 53,3 cubic feet, the content required.

EXAMPLE 2.

What is the content of a tree whoſe girt is 13 inches, and length is 40 feet 6 inches?

Now 13 × 13 - - = 169 the ſquare of the girt
The length - - = 40,5

845
676

The product - - = 6844,5, which, divided by 144, gives 47,5 the content required.

EXAMPLE 3.

What is the content of a piece of timber whoſe girt is 14 inches, and length is 20 feet? Now 14 × 14 = 196 the ſquare of the girt, and 196 × 20 = 3920 the product; which, divided by 144 gives 27,2 the content required.

Problem 37.

389. *The diameter of a globe being given to find the ſolidity, or content.*

RULES.

1. *Find the ſuperficies by the* 373, *multiply the ſuperficies by a third part of the ſemidiameter, and the product will give the content.*

Becauſe a globe is equal to a pyramid or cone, whoſe baſe is equal to the ſurface, and altitude is equal to the ſemidiameter - - - - - 270

2. *Find the content of a circumſcribing cylinder by the* 383, *and take two thirds of it for the content of the globe.*

For a globe is two thirds of its circumſcribing cylinder - - - - - - - 281

EXAMPLE.

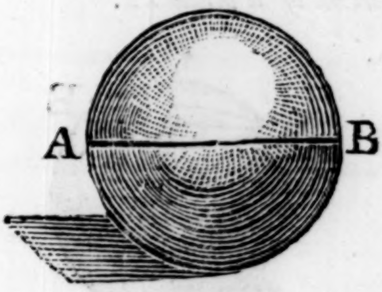
What is the content of a globe whoſe diameter is 1?

1. The ſuperficies - - - - = 3,1416
One third of the ſemidiameter - - = $\frac{1}{6}$
The content - - - - - - = 0,5236

2. The content of a circumſcribing cylinder is 0,7854, and $\frac{2}{3}$ of it is 0,5236, the content of the globe, as before.

Again, Problem 37.

390. *The diameter of a globe being given, to find the content.*



RULE.

Multiply the cube of the diameter by 0,5236, *and the product will give the content.*

For globes are proportional to the cubes of their diameters (by 291), and 0,5236 is the content of a globe whoſe diameter is 1 - - - - 389

EXAMPLE.

What is the content of a globe whoſe diameter AB is 12 inches?

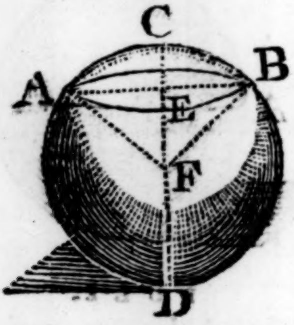
Now 12 × 12 × 12 - - = 1728 the cube of
diameter - - - 0,5236 ×

10368
5184
3456
8640

The content - - = 904,7808 cubic inches.

Problem 38.

391. *To find the content of a ſegment of a globe* ABC, *made by the ſection of a plain* AB.



RULE.

From three times the diameter of the globe, take twice the altitude of the ſegment; multiply together the remainder, the ſquare of the altitude, and 0,5236; *and this product will give the content.*

For the conical ſolid ACBF being equal to a pyramid of equal baſe and altitude (270) may be expreſſed in the given terms, and the cone ABF having the ſame baſe as the ſegment may likewiſe be expreſſed in the given terms, by what preceeds; and the difference of theſe expreſſions gives $\overline{3CD-2CE} \times CE^2 \times 0,5236$, the rule above.

Note. If the diameter of the ſegment AB is given, make CE : EA :: AE : ED, and CD will be known.

EXAMPLE.

What is the content of a ſegment 5 feet high, cut from a globe 20 feet diameter?

Now $60-10=50$ the remainder, and $5\times5=25=CE^2$.
Th. $50\times25\times0,5236=654,5$ the content required.

BRITISH
4 NO 73
MUSEUM

A TABLE *ſhewing the weight of a cubic foot, of various bodies, in pounds averdupoiſe.*

Things.	lb.	oz.
Gold	1227	8
Quickſilver	860	2
Lead	707	1
Silver	693	3
Copper	548	1
Steel	490	10
Bar iron	485	4
Block tin	452	6
Caſt iron	445	15
Marble	169	6
Portland ſtone	160	10
Freeſtone	147	0
Loom	135	0
Brick	125	0
Gravel	109	5
Clay	107	0
Chalk	67	7
Salt water	64	6
Rain water	62	8
Salt	43	12

Things.	lb.	oz.
Mahogany - - - - -	66 :	7
Oak - - - - - -	57 :	8
Beech - - - - - -	53 :	6
Aſh - - - - - - - -	50 :	0
Yellow fir - - - - -	41 :	1
Walnut - - - - - -	39 :	7
Elm - - - - - - -	37 :	8
White deal - - - - -	35 :	9
Peaſe - - - - - -	50 :	8
Beans - - - - - -	50 :	7
Wheat - - - - - -	48 :	4
Barley - - - - - -	41 :	2
Malt - - - - - -	30 :	5
Oats - - - - - -	29 :	8

BRITISH 4 NO 73 MUSEUM

FINIS.